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A NEW DECOUPLED FINITE ELEMENT ALGORITHM FOR VISCOELASTIC FLOW. PART 2: CONVERGENCE PROPERTIES OF THE ALGORITHM

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SUMMARY

We present the results of some numerical experiments which were carried out in order to investigate the general characteristics of the algorithm described in Part 1 of this paper.

KEY WORDS Finite elements Viscoelastic flow Convergence failure

1. INTRODUCTION

Part 1 of this paper 1 presented a decoupled finite element algorithm which the authors have developed for the simulation of steady two-dimensional viscoelastic flow. The algorithm incorporates non-consistent streamline upwinding and element subdivision for the stress field approximation. Results were given for two particular flow problems.

Before the results given in Part 1 were obtained, a series of numerical experiments were carried out in order to establish some general features of the behaviour of the decoupled algorithm. The version of the algorithm used for these experiments did not incorporate the element subdivision feature. Subsequent experience indicated that the properties demonstrated by these experiments remain valid when element subdivision is used.

The issues addressed in this paper are as follows.

(a) Manner of convergence and choice of convergence criterion. Convergence becomes more difficult as the Deborah number (*De*) is increased. In the range of values of *De* for which convergence occurs, exactly how does the error norm (defined below) vary? How is its manner of variation affected as *De* approaches the convergence limit? How do we decide when a solution can be considered to have 'converged'?

(b) Loss of positive definiteness of the matrix $\mathbf{T}_A$ (defined in Part 1). This accompanies loss of stability of the iterative scheme.

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(c) Effect of mesh refinement on ease of convergence for a simple flow geometry (planar parallel-sided half-channel).

2. FLOW PROBLEMS AND FINITE ELEMENT MESHES USED

Two flow problems were used for the numerical experiments: the straight planar parallel-sided half-channel and the abrupt planar 4:1 contraction. In each case the iterations begin from the Newtonian flow field, which is given as input data. The computations were performed with UCM model except where stated otherwise.

Figure 1 shows the meshes used for the straight half-channel problem. One side of the channel is an axis of symmetry and the other is a non-slip wall. The boundary conditions used for this and the 4:1 contraction are as discussed in Part 1. The first mesh in Figure 1 has four subdivisions across the width and five along its length and so is designated 4×5. The others are 4×10, 4×20, 8×10 and 12×10 respectively.

Figure 2 shows the mesh used for the 4:1 contraction problem. It has 144 elements.

3. DEFINITION OF THE ERROR NORM

The stress error norm in the nth iteration is given by

$$E_s=\frac{\Delta T^n}{|T|^n},$$

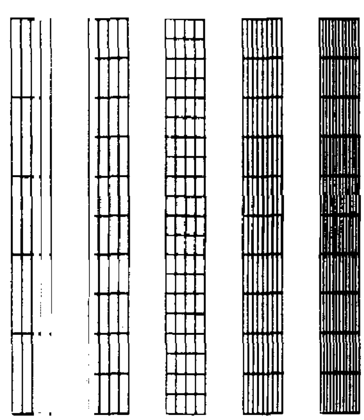

Figure 1. Straight channel

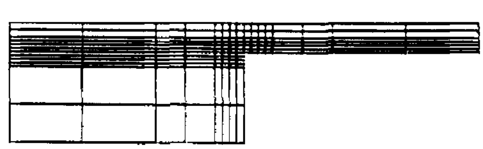

Figure 2. 4:1 contraction

where

$$\Delta T^n = \sum_I [(T^n_{xx_I} - T^{n-1}_{xx_I})^2 + (T^n_{xy_I} - T^{n-1}_{xy_I})^2 + (T^n_{yy_I} - T^{n-1}_{yy_I})^2],$$

$$|T|^n = \sum_I [(T^n_{xx_I})^2 + (T^n_{xy_I})^2 + (T^n_{yy_I})^2],$$

in which $T^n_{xx_I}$ is the value of T_{xx} at node I in the nth iteration, etc.

4. RESULTS

Manner of convergence; convergence criterion

In Figures 3 and 4 various examples are shown of the way in which the error norm varies as the iterations proceed. The results in Figure 3 were obtained with the 4×5 mesh, while Figure 4 is for the 8×10 mesh. In both cases upwinding was used with the upwinding scale factor ϕ equal to unity (i.e. so-called full upwinding). The general features seen in these results were also seen in the results obtained with the other straight channel meshes and with the 4:1 contraction. For sufficiently low values of *De*, as the iterations proceed, the error norm falls more or less monotonically to very small values. For sufficiently high values of *De* the error norm begins to increase after the first few iterations and the solution diverges.

In between these two extremes is a 'transition region' in which the solution cannot be said with any certainty to be either convergent or divergent. In some cases the error norm reaches a

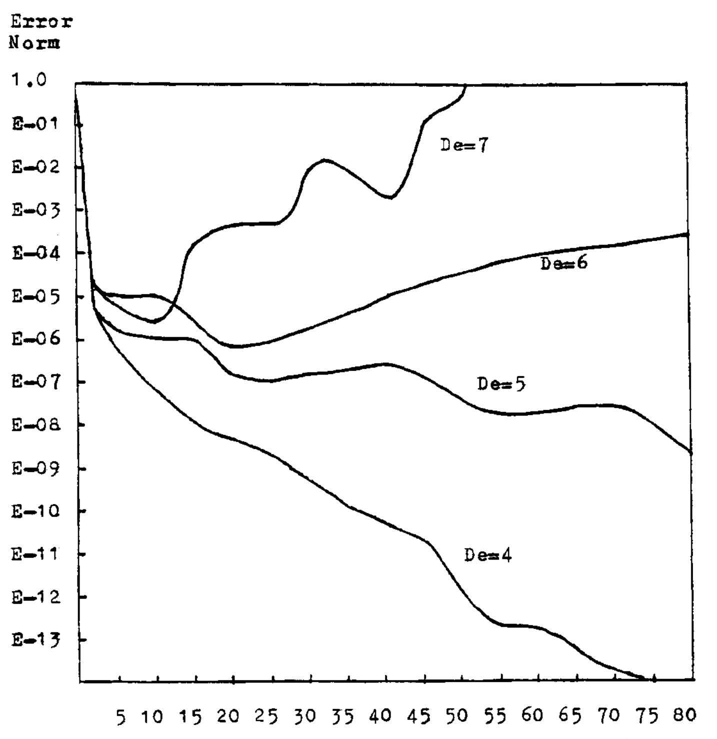

Figure 3. Stress error norm against iteration number for the straight channel 4×5 mesh with $\phi = 1$

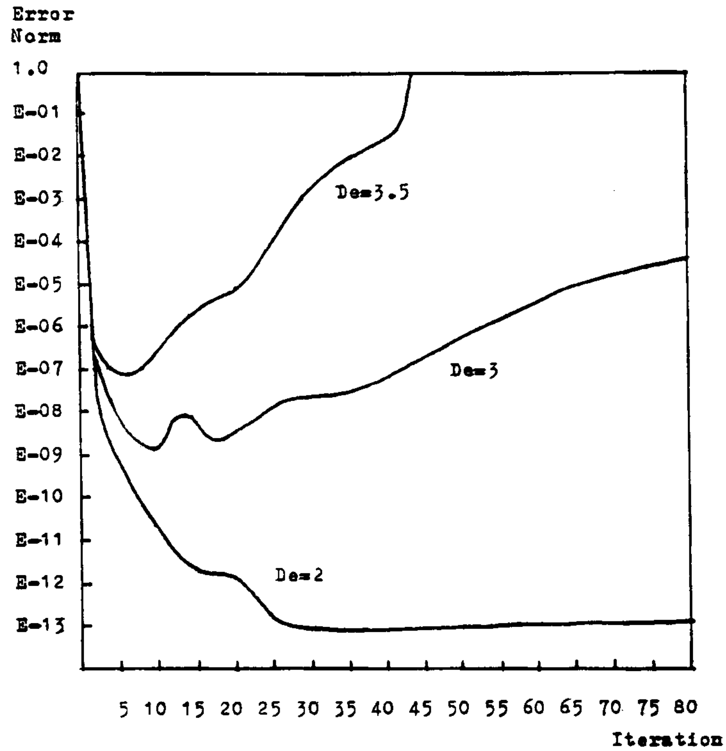

Figure 4. Stress error norm against iteration number for the straight channel 8 × 10 mesh with $\phi = 1$

minimum value, which can be very small, but thereafter increases so that the solution ultimately diverges.

The implication of these results is that as long as the maximum number of iterations allowed is restricted to a feasible value (within the limitations imposed by the available computing power), then the transition from convergence to divergence of the solution is not sharply defined. In practice it is necessary to choose a definite criterion to decide whether or not the solution is convergent—otherwise the computations would become impossibly expensive. The criterion which was used for the computations described in Part 1 was that the solution was considered to have converged if the error norm fell below a certain critical value within 100 iterations. The size of the critical value is a matter for compromise. The value we used was 10^{-4}; the results of the numerical experiments suggest that a larger value would not discriminate accurately enough between convergence and divergence, while a smaller value would have made the computations too expensive and time-consuming.

Loss of positive definiteness of the matrix $\mathbf{T}_A$.

The matrix $\mathbf{T}_A$ was defined in Part 1 as $\mathbf{T}_A = \mathbf{T} + (1 - r)\mathbf{I}/De$, where $\mathbf{T}$ is the Maxwell extra stress, r is the viscosity ratio and $\mathbf{I}$ is the unit matrix. Loss of convergence of the iterative scheme is associated with loss of positive definiteness of $\mathbf{T}_A$. Tables I and II illustrate the sort of behaviour which is observed when the solution is divergent. The number of mesh points at which $\mathbf{T}_A$ fails to be positive definite is denoted by N.

Table I. Straight half-channel (UCM); 12×10 mesh, $\phi=1$, $De=3{\cdot}5$

Iteration	Error	N
40	$2{\cdot}2\times10^{-2}$	0
41	$3{\cdot}3\times10^{-2}$	1
42	$3{\cdot}9\times10^{-2}$	4
43	$9{\cdot}0\times10^{-2}$	3
44	0·26	5
45	0·6	6
46	1·5	67

Table II. 4:1 contraction (Oldroyd-B); 144-element mesh, $\phi=1$, $De=0{\cdot}5$

Iteration	Error	N
2	$2{\cdot}3\times10^{-2}$	2
3	$7{\cdot}6\times10^{-2}$	2
4	$3{\cdot}2\times10^{-2}$	3
5	$6{\cdot}1\times10^{-2}$	2
6	$9{\cdot}1\times10^{-2}$	3
7	0·23	4
8	0·66	8
9	3·7	19

We can see that in these divergent cases the breakdown of the iterative scheme is associated with very rapid increases in N. Such behaviour was observed in all divergent cases. In cases for which the solution converges, N is always zero, except for cases close to the convergence limit when it may be one or two.

In the 4:1 contraction case the nodes at which $\mathbf{T}_A$ loses positive definiteness are generally in the vicinity of the re-entrant corner. As long as N is only two or three, these nodes are always very close to the corner and may include the node at the corner. As N increases, positive definiteness is lost at nodes lying farther from the corner, particularly on the downstream side of it.

Effect of mesh refinement in straight channel problem

Table III shows convergence limits for the different meshes used in the straight channel problem. Upwinding ($\phi=1$) was used in each case. The minimum value of the stress error norm in each case is also stated. The values of the De limits are accurate to $\pm0{\cdot}25$.

Table III. Straight channel

	4×5	4×10	4×20	8×10	12×10
De limit	6·0	2·0	1·0	3·0	3·0
Min. error	$2{\cdot}2\times10^{-6}$	$1{\cdot}0\times10^{-9}$	$2{\cdot}0\times10^{-17}$	$7{\cdot}0\times10^{-9}$	$7{\cdot}0\times10^{-10}$

Comparing the results for the 4×10, 8×10 and 12×10 meshes shows that refinement across the channel makes convergence easier. Comparing the results for the 4×5, 4×10 and 4×20 meshes shows that refinement along the channel length makes convergence more difficult.

For more complex flow geometries such as those studied in Part 1, the results have shown that in general mesh refinement makes convergence more difficult. It was also noted in Part 1 that improved mesh design for a given number of elements could make convergence easier.

For complex flow geometries other authors have also reported that mesh refinement makes convergence more difficult; examples are given in References 2 and 3.

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